

ELECTROMAGNETIC WAVE-PARTICLE WITH SPIN AND MAGNETIC MOMENT

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Abstract

An axisymmetric static solution of a nonlinear electrodynamics is considered as a massive charged particle with spin and magnetic moment. A linearization of the nonlinear electrodynamics around the static solution is investigated. The appropriate problem for linear waves around the static solution is considered. This wave part of the particle solution is considered to provide the appropriate wave properties for the particle. It has been found that the right (experimentally proved) formula for frequency of this wave appears theoretically for the static solution with ring singularity.

1 Introduction

The field electromagnetic particle concept in the framework of a unified nonlinear electrodynamics is considered here. In this approach a space-localized (soliton) solution of a nonlinear electrodynamics field model conforms to a physical elementary particle. The term “particle solution” will be used here. This theme was discussed in my articles (see, for example, [1, 2, 3, 4]). The present work is concerned mainly with possible existence of a quick-oscillating part for a particle solution in intrinsic coordinate system. This quick-oscillating part in intrinsic coordinate system of the particle will be a wave part for arbitrary coordinates. This wave part must provide the appropriate wave properties for the particle.

2 Static electromagnetic particle with spin

Axisymmetric static electromagnetic field configuration can have the spin defined as the full angular momentum of the electromagnetic field:

$$\mathbf{s} = \int \mathbf{M} dV, \quad (1)$$

where $\mathbf{M} \doteq \mathbf{r} \times \mathbf{P}$ is an angular momentum density (spin density), \mathbf{r} is a position vector, $\mathbf{P} \doteq (\mathbf{D} \times \mathbf{B})/4\pi$ is a momentum density (Poynting vector), \mathbf{D} and \mathbf{B} are electric and magnetic inductions.

The origin of the spin density is discussed in my articles. In particular, it was considered three topologically different static field configuration with spin [4]. These are configurations with two dyon, with singular disk, and with singular ring.

Let us consider more closely the field configuration with singular ring.

3 Toroidally symmetrical configuration with singular ring

A consideration of the static linear electrodynamics equations in toroidal coordinates (ξ, η) gives the appropriate solution with toroidal symmetry. This solution can include an electric and a magnetic parts. They can be represented with the help of toroidal harmonics which are the spheroidal harmonics with half-integer index: $P_{n-\frac{1}{2}}^l(\cosh \xi)$, where n and l are integer. To obtain the right behaviour of the electromagnetic field at infinity for a charged particle with magnetic moment we must take the toroidal harmonics $P_{-\frac{1}{2}}^0(\cosh \xi)$, $P_{-\frac{3}{2}}^0(\cosh \xi)$ for the electric field and $P_{-\frac{1}{2}}^1(\cosh \xi)$, $P_{-\frac{3}{2}}^1(\cosh \xi)$ for the magnetic one. Because we intend to consider this solution as an initial approximation to a solution of a nonlinear electrodynamics model, it is reasonable to take the condition of vanishing of two electromagnetic invariants near the singular ring. This condition will be satisfied when the ratio between the electric and magnetic vector magnitudes tends to unit near the ring.

We have the following appropriate solution of the linear electrodynamics:

$$\begin{aligned}
D_\xi &= -\frac{e}{\sqrt{2}\pi\rho^2} \sqrt{\cosh \xi - \cos \eta} \operatorname{csch} \xi \left[2(\cosh \xi - \cos \eta) \mathfrak{f}\left(-\sinh^2 \frac{\xi}{2}\right) \right. \\
&\quad \left. - (1 - \cos \eta)(1 + \cosh \xi) \mathfrak{k}\left(-\sinh^2 \frac{\xi}{2}\right) \right] , \\
D_\eta &= -\frac{e}{\sqrt{2}\pi\rho^2} \sqrt{\cosh \xi - \cos \eta} \mathfrak{k}\left(-\sinh^2 \frac{\xi}{2}\right) \sin \eta , \\
H_\xi &= -\frac{e}{\sqrt{2}\rho^2} i \sqrt{\cosh \xi - \cos \eta} P_{-\frac{1}{2}}^1(\cosh \xi) \sin \eta , \\
H_\eta &= \frac{e}{\sqrt{2}\rho^2} i \sqrt{\cosh \xi - \cos \eta} \operatorname{csch} \xi \left[(\cosh \xi - \cos \eta) P_{-\frac{3}{2}}^1(\cosh \xi) \right. \\
&\quad \left. + (\cos \eta \cosh \xi - 1) P_{-\frac{1}{2}}^1(\cosh \xi) \right] ,
\end{aligned} \tag{2}$$

where D_ξ , D_η , H_ξ , H_η are the physical components of the electric induction and magnetic strength vectors in toroidal coordinates, $\mathfrak{k}(m)$ and $\mathfrak{f}(m)$ are complete elliptic integrals of the first and second kinds accordingly (can be expressed by means of $P_{-\frac{1}{2}}^0(\cosh \xi)$ and $P_{-\frac{3}{2}}^0(\cosh \xi)$), $P_{-\frac{1}{2}}^l(z)$ and $P_{-\frac{3}{2}}^l(z)$ are associated Legendre functions with half-integer negative index. Here ρ is the radius of the ring and e is a constant.

The field configuration (2) has the electric charge e and the magnetic moment

$$\mu = \frac{e\rho}{2} . \tag{3}$$

About the definition of the electric charge and the magnetic moment see my article [3].

4 Wave part of the particle solution

In general case particle solution can include a time-periodic part. Having in view the wave properties of the physical particles this time-periodic part must be considered as quick-oscillating one. For the intrinsic coordinate system of the particle this quick-oscillating

part can be a standing wave or a progressive wave around a closed singular line (ring, for example). A hyperbolic rotation of the particle solution with the quick-oscillating part gives the solution for moving particle having evident wave properties.

The appropriate time-periodic solutions well known for the linear electrodynamics. In particular, they exist for considered topologically different configurations of singularities: bidyon, disk, and ring.

For the nonlinear case we can investigate the time-periodic part by means of the linearization of the problem around the appropriate static field configuration (which may not be exact solution). The general method for the linearization is based on Frechet derivative for the nonlinear operator:

$$\begin{array}{ccc} \mathcal{N}\mathbf{Y} = \mathbf{0} & \rightarrow & \mathcal{N}'(\mathbf{Y}_0)\tilde{\mathbf{Y}} = -\mathcal{N}\mathbf{Y}_0 \\ \text{Nonlinear} & & \text{Linearized around } \mathbf{Y}_0 \\ \text{system of equations} & \mathbf{Y} = \mathbf{Y}_0 + \tilde{\mathbf{Y}} & \text{system of equations} \end{array} .$$

This problem for point singularity was considered in my article [5].

5 About circular frequency of the time-periodic part of the particle solution

Let us consider that the circular frequency of the time-periodic part of the particle solution is given by the known formula: $m = \hbar\omega$ ($c = 1$).

The magnetic moment of the Dirac spin one-half particle is $\mu = \frac{e\hbar}{2m}$.

Thus we have the important formula for the circular frequency:

$$\left. \begin{array}{l} \mu = \frac{e\hbar}{2m} \Rightarrow \frac{m}{\hbar} = \frac{e}{2\mu} \\ m = \hbar\omega \Rightarrow \frac{m}{\hbar} = \omega \end{array} \right\} \Rightarrow \omega = \frac{e}{2\mu} \quad (4)$$

Here the initial formulas have the experimental confirmations. Thus the obtained formula must be considered as experimental one.

6 The advance for particle solution with ring singularity

The ring configuration gives birth to the appropriate periodic space boundary condition (on the ring). Thus it would appear reasonable that we will have the time-periodic part of the particle solution in the form of wave propagating along the ring. The wave-length of the appropriate fundamental mode is

$$\lambda = 2\pi\rho \quad . \quad (5)$$

In general case the static part of the solution can modify the conditions of propagation for the wave part of the solution (see my article [6]). But for the special case of vanishing of

the electromagnetic invariants for the static part of the solution we can have the changeless speed of light $c = 1$ for the wave part.

In this case the fundamental circular frequency for the time-periodic part is

$$\omega = \frac{2\pi}{\lambda} = \frac{1}{\rho} \quad . \quad (6)$$

Combining the formulas (3) and (6) we have:

$$\left. \begin{array}{l} \mu = \frac{e\rho}{2} \\ \omega = \frac{1}{\rho} \end{array} \right\} \Rightarrow \omega = \frac{e}{2\mu} \quad . \quad (7)$$

As we see this formula coincides with the experimental formula (4).

It should be noted that the appropriate field configuration with disk-shaped singularity (see, for example, [7]) has the magnetic moment described by formula $\mu = e\rho$ that is not leading in this approach to the right formula (4).

7 Conclusions

Thus we must call our attention to the particle solutions with ring singularity. Such particle solutions may represents the real physical elementary particles.

References

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